Modeling Induced Seismicity: Inter-Seismic Quasi-Static Triggering in A Discretely Fractured Poroelastic Medium

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ABSTRACT: We present a quasi-static and hydro-mechanical fully coupled approach to the modeling of inter-seismic triggering of seismicity in an arbitrary fractured and fluid-saturated poroelastic solid. The discrete fracture network (DFN) we consider is hybrid and at a dual-scale consisting of large-scale deterministic fractures and small-scale stochastic fractures following certain distributions. The fracture-poro-elasticity fully coupled modeling is carried out using our Jin & Zoback (2017) nonlinear computational model by explicitly resolving the deterministic fractures only. This provides inputs for the subsequent seismicity modeling that accounts for the entire hybrid dual-scale DFN. A new stress updating algorithm is developed accounting for the competing poroelastic stress compensation and seismicity-induced stress loss on fractures. Our model can therefore naturally produce multiple cycles of seismicity triggering. As an example, we perform a numerical experiment and generate a synthetic catalog of induced seismic events, and then analyze (1) seismicity distribution in relation to the fluid pressure, poroelastic stress and fracture distribution, (2) event type, (3) spatial-temporal characteristics, (4) stress history, (5) seismic source parameters and their characteristics and (6) activated DFN a scale consisting of large-scale deterministic fractures and small-scale stochastic fractures following certain distributions. The diffusion-based seismicity models can be further extended by incorporating, e.g., random stress heterogeneity (Goertz-Allmann & Wiemer, 2012), fractures following distributions derived from field observations (Verdon et al., 2015), and even empirical seismic emission criteria for generating synthetic seismograms (Carcione et al., 2015). This decoupled mechanism also underlies some studies that invert for distributions of permeability (Tarrahi & Jafarpour, 2012) and pore pressure (Terakawa et al., 2012; Terakawa, 2014) from induced seismicity data.

However, the decoupled mechanism inherently cannot explain the remoting triggering of seismicity in areas not subjected to pressure perturbation (Stark & Davis, 1996; Megies & Wassermann, 2014; Yeck et al., 2016); it also directly contradicts the commonly observed depletion-induced faulting (Zoback & Zinke, 2002). Motivated by such field evidences, a large body of analytical solutions (Segall, 1985; Segall et al., 1994; Segall & Fitzgerald,
1998; Altmann et al., 2014; Segall & Lu, 2015; Jin & Zoback, 2015a) and numerical solutions (Murphy et al., 2013; Chang & Segall, 2016a; Chang & Segall, 2016b; Chang & Segall, 2017; Fan et al., 2016; Deng et al., 2016; Zbiddin et al., 2017) have been proposed, providing poromechanics-based models of induced seismicity. At a smaller scale, numerical simulation of fluid-induced microseismicity, typically motivated by applications to stimulation of hydrocarbon and geothermal reservoirs, have been reported (e.g., Maillot et al., 1999; Baisch et al., 2010; Wassing et al., 2014; Zhao & Young, 2011; Yoon et al., 2014; Raziperchikolaee et al., 2014; Riffault et al., 2016). Irrespective of the scale of interest, these studies substantiate the critical role of poroelastic coupling in triggering seismicity. Despite these overwhelmingly strong evidences, some debates persist, mainly from those who advocate the simple diffusion-only models (Johann et al., 2016). They claim that their diffusion models approximate poroelastic models if the Biot coefficient \( \alpha \) is small; they also argue that when \( \alpha \) is less than 0.3, it is the pore pressure rather than the poroelastic stress that dominates the hydroseismicity on fractures; they even further question the Segall (2015, 2016a) poroelastic models in which \( \alpha \) is greater than 0.3, and simply hypothesize that for nearly impermeable rocks, the \( \alpha \) should also be negligible. However, one must realize that \( \alpha \) is a measurement of the rock solid’s susceptibility to the influence of the fluid and vice versa; it is not a property directly related to the permeability of the rock itself. As a matter of fact, laboratory experiments show that \( \alpha \) of unconventional reservoir rocks is indeed primarily between 0.3 and 0.9 (e.g., Ma & Zoback, 2017).

Fluid-solid coupling is undoubtedly important in triggering seismicity. However, the aforementioned poroelastic models only include fractures limited in both number and distribution, therefore, the role of fractures cannot be sufficiently explored; the fractures are also explicitly represented as entities following the same fluid and solid constitutive behaviors can significantly impact the outcome of modeled induced seismicity (Jin & Zoback, 2016a; Jin & Zoback, 2016b; Jin & Zoback, 2018c). Pertaining to this issue, some studies resolve very regularly distributed fractures (e.g., Safari & Ghassemi, 2016); others attempt to include an arbitrary DFN, among which, e.g., some focus on the fluid pressure and solid deformation only within fractures but not the matrix (Farmahini-Farahani & Ghassemi, 2016), some consider coupling only upon the occurrence of seismicity (Brue, 2007). None of these models produces repeating events frequently detected in fluid-induced seismicity catalogs (e.g., Baisch & Harjes, 2003; Moriya et al., 2003; Deichmann et al., 2014; Duverger et al, 2015).

To date, a model of induced seismicity considers fluid-solid full coupling in an arbitrarily fractured medium and at the same time capable of producing seismic source parameters and repeating seismic events is generally lacking. In this study, within the framework of single-phase poromechanics of arbitrarily fractured porous media developed by Jin & Zoback (2017), we first propose a coupled fracture-poro-elasticity model of induced seismicity in a fluid-saturated and discretely fractured porous solid. The model focuses on the inter-seismic triggering of seismicity and do not explicitly model the co-seismic rupture process. Seismicity-induced stress redistribution on a fracture is accomplished via a novel fracture stress updating algorithm. Further, a stochastic perturbation is introduced to the shear stress drop on a fracture induced to slip to allow for realistic seismic source characteristics. The model is also naturally capable of producing repeating seismic events. The details are described below.

2. METHODOLOGY

2.1. Two Competing Sources of Fault Stress

Given a location \( x \) and a time \( t \) over \( \Omega \times [0, T] \) where \( \Omega \) is the domain of interest and \([0, T]\) is the time interval, the effective stress tensor \( \sigma(x, t) \) in a fluid-saturated poroelastic medium undergoing seismic cycles can be decomposed as the following:

\[
\sigma(x, t) = \sigma_0(x) + \sigma_p(x, t) + \sum_j \sigma_j(x, t_j + \delta t_j)
\]  

(1)

where \( \sigma_0(x) \) is the initial in-situ effective stress tensor, \( \sigma_p(x, t) \) the fluid perturbation-induced poroelastic effective stress tensor relative to \( \sigma_0(x) \) and \( \sigma_j(x, t_j + \delta t_j) \) is the slip-induced change in the effective stress tensor over the \( j \)-th co-seismic interval where \( t_j^* \) and \( \delta t_j \) are the associated beginning time and duration. \( \sigma_0(x) \) is time-independent and in principle permits heterogeneity; \( \text{tr}(\sigma_j(x, t)) \) (the diagonal sum) is fully coupled with the gradient of the fluid pressure and the two must be solved for together; in \( \sigma_j(x, t_j + \delta t_j), \delta t_j \ll t \) such that relative to the time scale relevant to a complete seismic cycle, \( \delta t_j \approx 0 \) and \( \sigma_j(x, t_j + \delta t_j) \) can be approximated as a static quantity:

\[
\sigma_j(x, t_j + \delta t_j) \approx \sigma_j(x)
\]  

(2)

The stress on a fracture intersecting \( x \) and at time \( t \) is given by:

\[
\sigma_n(x, t) = \sigma(x, t) : n \otimes n
\]  

(3)

\[
\tau(x, t) = \left[ \left\| \sigma(x, t) : n \right\|^2 - \left( \sigma(x, t) : n \otimes n \right)^2 \right]^{1/2}
\]  

(4)
\[
CFF(x,t) = \tau(x,t) - \mu \sigma_n(x,t) \quad (5)
\]

In equations (3) - (5), \(\sigma_n(x,t)\) and \(\tau(x,t)\) are the effective normal stress, shear stress and Coulomb stress (i.e., the Coulomb Failure Function, \(\leq 0\)) on the fracture, and \(\mu\) and \(\mu_s\) are the unit normal vector and the static frictional coefficient of the fracture.

Equations (1) - (5) show that \(\sigma_p'(x,t)\) and \(\sigma_f'(x,t)\) are the two primary sources driving changes in the fault stress. In general, for \(\sigma_p'(x,t)\) and \(\sigma_f'(x,t)\) can either increase or decrease it whereas \(\sigma_f'(x)\) causes mostly negligible variations to it except near fracture tips; for \(\tau(x,t)\), \(\sigma_n'(x,t)\) compensates, albeit possibly negatively depending on the configuration, the fault for the shear stress loss resulting from \(\sigma_f'(x)\).

To model induced seismicity in a discretely fractured poroelastic medium, one must go through equations (1)-(5) and check \(CFF(x,t)\) against 0 to determine if seismicity occurs; if yes, the effective stress tensor needs to be updated \((j=j+1)\) for the next seismic cycle. This process needs to be repeated iteratively for all fractures at all time steps. For a given fracture that has undergone at least one seismic cycle, equations (1) - (4) yield a complete stress path associated with this cycle in the effective normal stress-shear stress space; the \(CFF\) remains constrained below 0 throughout the process.

The major computational cost then arises from the calculation of \(\sigma_n'(x,t)\) over the quasi-static inter-seismic (i.e., pre-seismic and post-seismic) phase and \(\sigma_n'(x,t)\) over the co-seismic phase. Notice these two processes can be modelled separately if assuming a linearly elastic solid irrespective of the fluid which can behave either linearly or nonlinearly. The former process can be sufficiently addressed using our Jin & Zoback (2017) computational model; for a detailed description on the latter process resulting from a fully dynamic and spontaneously rupturing seismic event while considering the effect of \(\sigma_p'(x,t)\), we refer the readers to Jin & Zoback (2018a, 2018b). In this study, we are concerned only with the inter-seismic evolution of induced seismicity but not the co-seismic dynamic changes (i.e., wave propagation), therefore, instead of solving for both \(\sigma_p'(x,t)\) and \(\sigma_p'(x,t)\) for updating the fracture stress, we will instead solve only for \(\sigma_p'(x,t)\) and then insert it into a stress updating algorithm to indirectly account for seismicity-induced stress changes on a fracture. The details of these two steps are given in the following two sections.

### 2.2. Coupled Fracture-Poro-Mechanical Modeling

The objective here is to calculate \(\sigma_p'(x,t)\) as an input for updating the fracture stress. Within the framework of Biot's theory of poroelasticity, Jin & Zoback (2017) formulated the problem of fluid-solid fully coupled quasi-static poromechanics of an arbitrarily fractured deformable porous solid saturated with a single-phase compressible fluid. Without attempting to present the full details, here, we outline several key governing equations.

First, the fully coupled mass conservation law and quasi-static force balance law are:

\[
\begin{align*}
\phi_{sof}(x)(C_m + C_p)(1 - \Lambda_g(x))\phi_{sf}(x)(C_f + C_p) - \alpha \nabla \cdot \dot{u}(x,t) + \nabla \cdot \nu(x,t) = s(x,t), \quad x \in \Omega_m \cup \Omega_f \\
\nabla \cdot \sigma_p'(x,t) + \alpha \nabla p(x,t) = 0, \quad x \in \Omega_f
\end{align*}
\]

Next, the two fluid flow equations are given by the Darcy's law and a nonlinear cubic law, designated to the matrix and fractures, respectively. They read:

\[
\begin{align*}
\nu(x,t) &= -\eta \nabla \cdot \nu(x,t), \quad x \in \Omega_m \\
\nu(x,t) &= -\eta \frac{1}{12} b_0 (1 + C_f \mu_f(x,t)) \nabla \cdot \nu(x,t), \quad x \in \Omega_f
\end{align*}
\]

Furthermore, the two solid constitutive laws, including a generalized Hooke's law for the matrix and a transverse simple shear deformation law for fractures, read:

\[
\begin{align*}
\sigma_p'(x,t) &= C_m : \nabla \nu_m(x,t), \quad x \in \Omega_m \\
\sigma_p'(x,t) &= G_f \nabla \mu_f(x,t), \quad x \in \Omega_f
\end{align*}
\]

In equations (6) - (11), subscripts ‘\(m\)’ and ‘\(f\)’ indicate quantities associated with the porous matrix and discrete fractures, subscript ‘\(0\)’ denotes the initial value of a quantity, subscripts ‘\(n\)’ and ‘\(\tau\)’ indicate the fracture normal and tangential directions, \(x\) and \(\xi\) indicate the global and fracture local coordinate systems, \(t\) is the time, \(\Omega\) is the model domain, \(\phi\) is the intrinsic porosity, \(\Lambda(x)\) is a fracture-dependent parameter enabling the definition of a so-called partial porosity, \(C\) is the compressibility, \(p\) is the fluid overpressure, \(\nu\) is the fluid velocity vector, \(s\) is the external fluid source normalized by the initial fluid density, \(\eta\) is the fluid viscosity, \(k\) is the permeability tensor, \(b\) is the fracture hydraulic aperture, \(\sigma_p\) is the solid effective stress (i.e., poroelastic stress) tensor, \(u\) is the solid displacement vector, \(\alpha\) is the Biot coefficient, \(I\) is the Kronecker delta, \(C\) is the elastic stiffness tensor under a plane strain assumption, \(G\) is the fracture shear modulus. Note that \(\nabla, \nabla^n, \nabla_n\) and \(\nabla_\tau\) are operators for computing the gradient, the symmetric gradient, the fracture-normal gradient and the fracture-tangential gradient, and \(\nabla \cdot\) is the divergence operator.

The initial conditions of the primary unknowns are trivially set up as 0 since we are solving only for the relative changes. Standard Dirichlet and Neumann boundary conditions are prescribed for both the fluid and the solid. Additionally, a fluid interface condition is prescribed over fractures of a certain type. Details are given in Jin & Zoback (2017).
The presence of fractures is reflected in equation (6) by the modification to the hydraulic storage capacity, and by equations (9) and (11) as the augmentation to the hydraulic conductivity and the elastic stiffness of the system. Fracture-induced nonlinearity is introduced by equation (9) via the pressure-dependent hydraulic aperture. Additionally, by formulating the problem over a single domain, the mass exchange between fractures and the matrix is resolved by, (1) imposing an interface condition in addition to the standard Dirichlet and Neumann boundary conditions, and (2) admitting discontinuities in fracture-normal fluid flux. We note that our model is different from the standard dual-porosity double-permeability model which requires the formulation of two interacting mass conservation laws and the use of a smearing quantity called the ‘shape factor’ resulting from domain separation and regularization.

Jin & Zoback (2017) developed a hybrid-dimensional two-field mixed finite element method for efficient space discretization of the above fully coupled problem, leading to the following semi-discrete form:

\[
\begin{align*}
& \left[ \mathbf{M}_m + \sum_j \mathbf{M}_f (\hat{\xi}_j) - \mathbf{C}^T \right] \left[ \hat{\xi}_m \right] + \\
& \left[ \mathbf{K}_m + \sum_j \mathbf{K}_f (\hat{\xi}_j) - \sum_k \mathbf{K}_{mf} (\hat{\xi}_j) \right] \mathbf{Y} + \sum_k \mathbf{K}_{fm} (\hat{\xi}_j) + \sum_k \mathbf{K}_{fmw} = 0 \\
& \mathbf{G}_m + \sum_i \mathbf{G}_{fi} (\hat{\xi}_j) \end{align*}
\]

\[= \left\{ \mathbf{F} \right\} \quad \left\{ \mathbf{Y} \right\}
\]

(12)

where \( \mathbf{M} \) is the fluid storage capacity matrix, \( \mathbf{K} \) is the hydraulic conductivity/transferability matrix, \( \mathbf{G} \) is the stiffness matrix, \( \mathbf{C} \) is the coupling matrix, \( \mathbf{F} \) is the external nodal mass, \( \mathbf{Y} \) is the external nodal force, \( \hat{\xi} \) and \( \hat{d} \) are the nodal fluid pressure and solid displacement vectors, subscripts ‘m’ and ‘f’ are as before, subscripts ‘mf’ and ‘fm’ indicate matrix-to-fracture and fracture-to-matrix interactions, and \( i, j \) and \( k \) indicate the index of a fracture of a certain type.

Equation (12) retains a canonical form of that resulting from the space discretization of a standard poromechanical problem except for the modification and augmentation to \( \mathbf{M}, \mathbf{K} \) and \( \mathbf{G} \) due to fractures. Equation (12) is then discretized in time in a fully coupled manner (as opposed to a sequential manner) using the backward Euler scheme and is further linearized using the Newton-Raphson scheme. Details can be found in Jin & Zoback (2017). Solving the resulting fully discrete form for each time step yields the input, specifically, \( \mathbf{r}_0 (\mathbf{x}, t) \), for the subsequent modelling of induced seismicity.

### 2.3. Fracture Stress Updating Algorithm and Seismicity Modeling

The objective here is to update the fracture stress resulting from both \( \mathbf{r}_0 (\mathbf{x}, t) \) and \( \mathbf{r}_s (\mathbf{x}, t) \). For the reasons explained in section 2.1, we will update the fracture stress first using only \( \mathbf{r}_0 (\mathbf{x}, t) \) and then correct for changes due to \( \mathbf{r}_s (\mathbf{x}, t) \). To do so, we make two assumptions. First, fracture slip causes negligible changes in the effective normal stress on the fracture. This is a reasonable assumption for the area on the fracture not immediately near its tips. From equation (3), this reads:

\[
\mathbf{r}_s (\mathbf{x}, t) : \mathbf{n} \otimes \mathbf{n} \approx 0
\]

(13)

Equation (13) implies that,

\[
\left( \mathbf{r}_0 (\mathbf{x}, t) + \mathbf{r}_s (\mathbf{x}, t) + \sum_j \mathbf{r}_s (\mathbf{x}, t) \right) : \mathbf{n} \otimes \mathbf{n} \approx \left( \mathbf{r}_0 (\mathbf{x}, t) + \mathbf{r}_s (\mathbf{x}, t) \right) : \mathbf{n} \otimes \mathbf{n}
\]

(14)

On the other hand, the shear stress on the fracture stated by equation (4), when accounting for the effect of \( \mathbf{r}_0 (\mathbf{x}, t) \), can be re-written in the following form:

\[
\left( \mathbf{r}_0 (\mathbf{x}, t) + \mathbf{r}_s (\mathbf{x}, t) + \sum_j \mathbf{r}_s (\mathbf{x}, t) \right) : \mathbf{n} \otimes \mathbf{n}^2
\]

(15)

\[
= \left( \mathbf{r}_0 (\mathbf{x}, t) + \mathbf{r}_s (\mathbf{x}, t) \right) : \mathbf{n} \otimes \mathbf{n} \approx \sum_j \Delta \tau_j
\]

Here, \( \Delta \tau_j \) is the shear stress drop due to the \( j \)th co-seismic interval. Our second assumption reads:

\[
\Delta \tau_j = r (\mu_d - \mu_s) \left( \mathbf{r}_0 (\mathbf{x}, t) + \mathbf{r}_s (\mathbf{x}, t) \right) : \mathbf{n} \otimes \mathbf{n}
\]

(16)

where \( \mu_d \) is the dynamic frictional coefficient of the fracture as is typically used in a slip-weakening law (Andrews, 1976), and \( r \) is a random perturbation parameter between 0 and 1 to account for the potential non-full degree of shear stress drop (see also Verdon et al., 2015). In this study, we let the probability density function of \( r \) follow a uniform distribution. Equation (16) states that (1) the new shear stress on a fracture following seismicity is constrained above a lower bound defined by the residual frictional strength of the fracture and (2), more importantly, the shear stress drop is dictated by the evolution of the poroelastic stress. This is fundamentally different from directly prescribing the shear stress drop (e.g., Izadi & Elsworth, 2014).

Based on the above two assumptions, we propose the following incremental fracture stress updating algorithm, as is shown in List 1.
The above algorithm automatically considers multiple seismic cycles and therefore is capable of modeling repeating seismic events (i.e., doublets and multiplets). We are now at a place to model the inter-seismic triggering of induced seismicity in a discretely fractured poroelastic medium using the above quasi-static and fracture-poro-elasticity fully coupled approach accounting for seismicity-induced stress redistribution on slipped fractures. A complete seismic event catalog containing information on, e.g., event origin time $t_0$, location $\mathbf{x}$, shear stress drop $\Delta \tau$, seismic moment $M_0$, moment magnitude $M_{W}$, fracture length $L$, initial Coulomb stress $\text{CFF}_0$ and permeability changes, can be assembled. Several key equations used in calculating these source parameters are shown in the appendix.

3. MODEL SET-UP

As a numerical example, we construct a 200 m × 200 m 2D domain representing a porous rock hosting a large number of discrete fractures, see Figure 1. To render this exercise numerically more tractable, we propose to resolve only the large-scale fractures (large relative to the domain size) in the fully coupled fracture-poro-mechanical modeling to model the evolution of $\sigma_p'(x, t)$. Small-scale fractures will be stochastically represented; they do not contribute to the modeling of $\sigma_p'(x, t)$ but nevertheless will participate in the seismicity modeling and stress updating shown in section 2.3. Suppose the large-scale DFN consisting of 100 fractures in Figure 1 is a deterministic representation of real data, with the fracture length ranging from 20 m to 50 m, and the orientation, from 0 to 360°. This fractured model domain is then discretized in space (see Figure 2) as a step in arriving at the semi-discrete form equation (12). For simplicity, here we only solve equations (6) - (10) but not equation (11), and the contribution of the DFN to the stiffness matrix $\mathbf{G}$ is therefore left out. For a more comprehensive numerical example, see Jin & Zoback (2017). Some nominal mechanical parameters are: $\sigma_0'=[15.0; 0.505]$ MPa, $\mu_s=0.6, \mu_d=0.4$, and $\lambda=\mu=16$ GPa; the hydraulic properties, fluid and solid boundary values and time-stepping parameters are identical to those in Jin & Zoback (2017). A plane strain condition is assumed.

Figure 1. The model domain used for modeling the fully coupled poroelastic problem. It consists of a large-scale deterministic discrete fracture network embedded within an otherwise porous matrix. Dots represent centers of the fractures, colored with their assigned indices within the fracture population. Magenta and blue lines represent interconnected and isolated fractures respectively in relation to the fluid.
boundary condition (or external fluid source) as are indicated by the purple circle and the dark red lines; they require different treatment of the mass exchange with the surrounding matrix, see Jin & Zoback (2017) for details.

All elements represent the porous matrix. The grey elements are standard two-field (fluid pressure, solid displacement) mixed FE elements; the colored elements are ‘hybrid’ mixed elements in which at least one edge is also used as a lower-dimensional element to discretize fractures. The color of an element indicates the fractures with which it is associated. If a hybrid element conforms to multiple fractures, only the largest index is used for coloring. Adapted from Jin & Zoback (2017).

As an example, a stochastic small-scale DFN is generated following a realistic distribution while honoring existing information. For simplicity and this does not change the generality of our method, we let the total number of fractures be equal to that of finite elements; within the fracture-conforming elements, the fracture orientation coincides with that of the corresponding deterministic fractures; within the rest elements, the fractures orientation follows a uniform distribution on [0, 360°]; the fracture length is generated by obeying a realistic power-law distribution as is observed in fracture data (e.g., Johri & Zoback, 2014) and even microseismic data (Jin & Zoback, 2015b), and is randomly distributed to all fractures. Figure 3 illustrates this process.

Figure 3. The discrete fracture network used for seismicity modeling and stress updating. A fracture is reassigned to the center of each element shown in Figure 2. (a) Within any hybrid element, the fracture orientation honors that of the corresponding large-scale deterministic fracture, (b) within the rest elements, the fracture orientation is randomly generated between 0 and 360° following a uniform distribution, and (c) the resulting hybrid deterministic-stochastic dual fracture network. In Figures 3a-3c, the color indicates the susceptibility of a fracture to slip under the given initial in-situ stress state; warm and cool colors indicate favorable and unfavorable orientations, respectively; the fracture length follows a realistic power-law distribution (details shown later in Figure 13) and is randomly assigned to all 12800 fractures.

Figure 4 shows the initial effective normal stress and shear stress on all fractures, and the color indicates the associated CFF. The model domain is critically stressed. The red and green lines indicate the peak and residual frictional strengths of all fractures, calculated from $\mu_s$ and $\mu_d$, respectively. The peak strength defines the upper bound of the fracture shear stress; if seismicity occurs at certain effective normal stress, the corresponding difference between the peak strength and the residual strength defines the maximum likely shear stress drop (i.e., the residual strength defines the lower bound to which seismicity can bring the shear stress).
4. RESULTS

4.1. Fluid Pressure, Poroelastic Stress and Seismicity

Figures 5 - 8 are four snapshots of the distribution of modeled changes in several quantities, including (1) the fluid overpressure \( p \), (2) the first stress invariant \( I_1' / 3 \), (3) the second deviatoric stress invariant \( J_2' \) ((2) and (3) are calculated from the effective poroelastic stress tensor \( \sigma_p' \) under plane strain), (4) the excess poroelastic shear stress, defined as \( J_2' - \sin(\phi) I_1' / 3 \) where \( \phi = \tan^{-1}(\mu_s) \) is the frictional angle and is assumed to be the same for all fractures (adapted from Borja, 2013) and (5) the associated seismicity sized with \( M_w \) and colored with \( \Delta \tau \).

In all plots, only the 100 m x 100 m area around the center is shown. All modeled quantities are heavily controlled by the distribution of the large-scale deterministic DFN, precluding the definition of a smooth ‘front’ (e.g., pressure front, seismicity front). The fluid pressure increases primarily along fractures and secondarily within the matrix. Poroelastic normal stress develops, dominantly being extensional near the fractures. Notice how the area subjected to the poroelastic normal stress is beyond that under the fluid pressure alteration. A pronounced shear stress field also develops and influences a large portion of the domain. These observations are due to the fluid-to-solid coupling. Although not shown here, the solid-to-fluid coupling can also be observed when plotting the fluid pressure on a logarithmic scale (see Jin & Zoback, 2017). Unlike the results produced by the diffusion-only class of models (Shapiro et al., 2005), the seismicity distribution resulting from our model is not only highly heterogenous but also clustered near certain fractures, as is frequently corroborated by field observations (e.g., Baisch & Harjes, 2003; Stabile et al., 2014; Deichmann et al., 2014; Block et al., 2015). Interestingly, the distribution of seismicity shows no direct correlation with that of \( p, I_1' / 3 \), \( J_2' \). However, by further examining the fracture orientation (Figure 3), it becomes clear that seismicity is clustered only near fractures that are well-oriented or sub-well-oriented with respect to \( \sigma_0' \) and at the same time subjected to the excess poroelastic shear stress. Furthermore, the delineated seismicity front is always within the stress front. This is because while the domain is overall critically stressed, the critical state occurs only at fractures of a particular orientation; for the remaining fractures of various other orientations, a sufficient amount of stress needs to be generated before triggering seismicity. Finally, \( M_w \) and \( \Delta \tau \) exhibit no spatial or temporal dependence, and no correlation is observed between \( M_w \) and \( \Delta \tau \) (all can be done through simple cross plots, details are omitted) due to the random fracture length. Our modeling therefore highlights the critical importance of accounting for the interaction among fractures (distribution, orientation and length), the initial stress state and the poroelastic coupling.
Figure 5. Snapshots of four modeled quantities 10 minutes since the injection. (a) The fluid pressure $p$, (b) the poroelastic effective mean normal stress $I_1^{1/3}$, (c) the second deviatoric poroelastic stress invariant $\sqrt{J_2'}$, (d) the excess poroelastic shear stress, defined as $\sqrt{J_2'} \cdot \sin(\phi) I_1^{1/3}$ where $\phi = \tan^{-1}(\mu_s)$ and (e) the seismic events sized by moment magnitude $M_w$ and colored by stress drop $\Delta \tau$.

Figure 6. Same as Figure 5 except the time is 20 minutes since the injection.

Figure 7. Same as Figure 5 except the time is 40 minutes since the injection.
4.2. Event Classification

The induced seismic events can be categorized into different groups, as is shown in Figures 9a-9f. In all plots, the events are again sized with $M_w$ but now colored with the event origin time $t_0$. Specifically, Figure 9a shows the spatial-temporal distribution of all 875 events. Thanks to the idea of incorporating poroelastic stress into seismic cycles, our model can naturally produce repeating events, as are shown in Figure 9b. Each location shown indicates a doublet or a multiplet group (e.g., Poupinet & Ellsworth, 1984; Waldhauser & Ellsworth, 2002) which contains two or more events that occur on the same source location but at different time; for visibility, a small-magnitude event is always plotted within a big-magnitude one (see the concentric circles). A total of 85 doublet or multiplet groups containing 178 events are modeled, see also Figure 10. Here, two observations can be made. First, in general, repeating events are concentrated in areas where the event density is high. From the analysis provided in section 4.1, this implies that repeating events are favored in areas with more favorably-oriented fractures and under the excess shear stress simultaneously. Second, within each group, an earlier event does not necessarily have a larger magnitude; the contrary can be observed for certain groups. This is due to the complex stress path (section 4.4) and the non-full degree of stress drop ($r$ in equation (16)).

Figures 9c and 9d show the so-called explicit and implicit events, respectively. We refer to an explicit event as one occurring on a fracture with deterministic location and orientation (Figure 3a) while an implicit event as one associated with a stochastic fracture (Figure 3b). In Figure 9c, the explicit events clearly depict lineations in alignment with the well-oriented subset of deterministic fractures. In general, the along-fracture distance of an explicit event correlates positively with its origin time. This is due to that within each group of deterministic fractures, the orientation is identical and the required excess shear stress is the same, therefore, the progressive increase in the excess poroelastic shear stress along each group (Figures 5-8) leads to the progressive development of the events (as will also been shown in Figure 11f). For the implicit events in Figure 9d, however, this trend immediately breaks down for the very same reason: the presence of the deterministic fractures and the associated heterogeneity in the excess poroelastic stress, when acting on stochastic fractures of various orientations, lead to the random spatial-temporal evolution of these events within the matrix.

Finally, Figures 9e and 9f show the so-called wet and dry events (Maxwell et al., 2015), which qualitatively speaking, occur at locations with and without significant fluid pressure perturbation, respectively. The threshold value used here for plotting is 10% of the injection pressure. Traditionally, the triggering of these two types of event are attributed to the fluid overpressure and the mechanical stress transfer, respectively. However, we must point out that in a poroelastic medium, the triggering mechanism should always be the excess poroelastic shear stress rather than the fluid pressure. In that sense, the discrimination between wet and dry events is less meaningful. For example, while the great majority of the events here are wet as opposed to dry, we have demonstrated in Figures 5-8 that the distribution of the events correlates not with the fluid overpressure but the excess poroelastic shear stress (in conjunction with the presence of well-oriented and sub-well-oriented fractures). The fact that most events are wet is simply due
to that the area subjected to the excess poroelastic shear stress sufficient enough to trigger seismicity also undergoes pronounced fluid pressure changes simultaneously.

The modeling outcome as is illustrated through Figures 9a to 9d are commonly observed in real data, therefore demonstrating the advantage of our model towards the simple fracture-free and coupling-free fluid-diffusion models that inherently cannot produced none of the above observations.

Figure 9. Classification of induced seismic events, sized by the moment magnitude $M_w$ and colored by the event origin time $t_0$. The number of events are indicated on the top left in each plot. (a) All events, (b) repeating events, (c) explicit (deterministic) events, (d) implicit (stochastic) events, (e) wet events, and (f) dry events.
4.3. **R-T Characteristics**

The spatial-temporal characteristics of the modeled quantities are further illustrated using the so-called R-T (R is the distance and T is the time) plot shown in Figure 11. In all plots, the distance R is calculated with respect to the domain center and is plotted only from 0 to 45 m. The fluid overpressure $p$ is shown in Figure 11a, overlaid with six iso-diffusivity profiles calculated as $R = \sqrt{4\pi DT}$ where $D$ is the effective hydraulic diffusivity of the medium; this is an expression derived from a linear diffusion process resulting from a point source injection in an isotropic, homogeneous and porous only medium free from poroelastic effect, and is frequently referred to as the seismicity triggering front (Shapiro et al., 1997; Shapiro et al., 2002). The green and magenta lines correspond to the diffusivity of the matrix and fractures, respectively. Notice how the delineated pressure front lies between the two and is approximately near the profile corresponding to a 0.03 m$^2$/s diffusivity. Figure 11b shows the excess poroelastic shear stress $\sqrt{\overline{J}_2} \cdot \sin(\phi) M_1^{1/3}$, where the stress front exceeds that of the pressure (see also Figures 5-8). Notice in Figures 11a and 11b, for better visibility, the color scale is flipped compared to that used in Figures 5-8. We also point out that the term pressure/stress front is only used qualitatively and it refers to where and when noticeable changes in the overpressure/poroelastic stress can be observed in the R-T space.

Figures 11c-11f are the R-T distribution of seismicity. Specifically, Figures 11c shows all seismic events sized with $M_w$ and colored with the initial Coulomb stress $CFF_0$ of the associated fracture. In a different way shown in section 4.1, here Figures 11c illustrate that (1) the seismicity front is below the pressure front, which is further below the stress front, (2) the distribution is highly heterogeneous due to the fractures, and (3) even under the poroelastic stress, the majority of the events occur primarily on well-oriented fractures, some on sub-well-oriented fractures and only a few on poorly-oriented ones. In the context of unconventional reservoir stimulation, this implies that fracture orientation with respect to the initial stress field is a first-order controlling factor for stimulation efficiency. Figure 11d resembles Figure 11c except for the color which shows the shear stress drop, which exhibits no spatial-temporal dependence. The distribution of repeating events is plotted in Figure 11e, which illustrates the ‘breakdown’ of the parabolic shape of the seismicity front. Finally, Figure 11f singles out the explicit events, and the color indicates the fracture with which an event is associated. The progressive development of events along a particular (deterministic) fracture, as has also been explained in section 4.2, now becomes evident, i.e., events of the same color delineate a parabolic trend. Implicit events show no such characteristic.
Figure 11. Space-time plot of modeled quantities. (a) The fluid pressure, (b) the excess poroelastic shear stress, (c)-(d) all seismic events, (e) repeating events and (f) explicit events. In each plot, the distance is calculated with respect to the origin (0,0); the green and magenta lines indicate the characteristic diffusion profiles calculated using the diffusivity of the matrix and the fractures, respectively; four additional profiles are also shown together with the corresponding diffusivities. In Figures 11c-11f, the events are sized with Mw, and the color indicates CFF0, Δτ or the fracture index.

4.4. Stress Path

Figure 12. Effective normal stress-shear stress on all fractures that have undergone failure at least once (i.e., one seismic cycle). The color indicates the CFF. (a) The initial reference state showing the failed fractures sampling a subset of all fractures shown in Figure
Figure 12 shows the stress paths (i.e., stress history) of all fractures that have undergone failure (including repeated failure). Figure 12a shows the initial reference stress state. The locations of the fractures on an otherwise complete Mohr circle shows that it is mostly the well-oriented and sub-well-oriented fractures that are induced to slip. Figures 12b-12d are snapshots of the intermediate stress state at three time steps. For any fracture, the arrow indicates the change relative to the initial state. In contrast to a simple leftward translation predicted by a decoupled approach, here the poroelastic stress compensation leads to a bended stress path (within each seismic cycle). The stress state remains constrained below the peak strength (the red line); when the \( CFF \) reaches 0, the shear stress drop is enforced and the new stress state is bounded from below by the residual strength (the green line). This stress compensation-drop process is repeated for fractures generating doublets and multiplets (Figures 9b, 10). Poroelastic coupling plays a critical role in this process, as it dictates the location on the red line where a fracture reaches failure (see equation (16)), hence, the stress drop and the other seismic source parameters.

4.5. Magnitude-Frequency Relation

It is well established that the number of fractures within a natural fracture system scales with the fracture length according to the following power law (e.g., Bonnet et al., 2001):

\[
N = CL^{-D}
\]  

(17)

where \( N \) is the number of fractures of length \( L \), \( C \) is a site-specific constant and \( D \) is the so-called fractal dimension.

On the other hand, earthquakes in nature are characterized with a universal statistical relation between the magnitude and the frequency, namely the Gutenberg-Richter law (Gutenberg, 1956), which reads:

\[
\log N(m > M_w) = a - bM_w
\]  

(18)

where \( N(m > M_w) \) is the total number of events with a moment magnitude \( m \) above \( M_w \), and \( a \) and \( b \) are constants.

In nature, \( D \) is frequently observed to be between 1 and 2 (e.g., Okubo & Aki, 1987), whereas a common value of \( b \) is around 1 (e.g., Shi & Bolt, 1982). Studies suggest that \( D \) and \( b \) are inherently related. For example, Hirata (1989) suggests that \( D \approx 2b \). What is somewhat curious is that for induced seismic events, \( b \) is often above 1 (e.g., Vermylen & Zoback, 2011) and a frequently observed value is around 2 (e.g., Bachmann et al., 2012).

Here in Figure 14, the fracture length distribution as is discussed in section 3 is plotted (green), together with the power law fitting line (magenta); the length distribution of the shear re-activated subset of fractures is also shown (red), which no longer obeys the power law decay, owing to that only favorably oriented fractures are induced to slip. Nonetheless, the magnitude-frequency relation still holds for the modeled induced events, as is illustrated in Figure 15. The distribution of \( M_w \), which primarily vary between -3.5 and -1.0, is shown as the histogram (yellow green); the total number of events (i.e., cumulative frequency) is shown by the blue green dots, which is then used to fit the Gutenberg-Richter law, yielding \( b = 1.987 \). We hypothesize that the breaking-down in the power law distribution of the length of the activated subset of fractures might be responsible for the deviation in the \( b \)-value for induced seismicity.
distribution of $N$ follows the classic Gutenberg-Richter law (blue green); data points with a $M_w$ above -2 are used for fitting (the magenta line), yielding a $b$-value around 2, which is commonly observed for induced seismicity.

4.6. Activated DFN and Permeability Enhancement

Figure 16 demonstrates the growth of the activated fracture network; the color indicates the associated changes in the fracture normal and tangential permeabilities, calculated using equation (21). The activated network includes fractures both interconnected to and isolated from the fluid boundary, due to the complex poroelastic stress distribution with respect to the DFN. In the context of unconventional and geothermal reservoir stimulation, our modeling provides a comprehensive way to predict the so-called stimulated reservoir volume (SRV) and the stimulation efficiency.

![Figure 16. Snapshots of the activated fractures at six selected time steps showing the progressive development of the stimulated network. The time is indicated at the top of each plot and the color indicates the nondimensionalized permeability changes along the fracture normal and tangential directions.](image)

5. SUMMARY AND CONCLUSION

We have developed a general and physics-based method for the modeling of inter-seismic quasi-static triggering of seismicity in a fluid-saturated and discretely fractured poroelastic media. The method is casted within the framework of single-phase and fluid-solid fully coupled poromechanics of arbitrarily fractured porous media developed by Jin & Zoback (2017), with the addition of a fracture-stress updating algorithm that allows for the consideration of seismicity-induced stress changes on slipped fractures without explicitly resolving the coseismic rupture process. The DFN we consider is dual-scale and hybrid, consisting of large-scale deterministic fractures and small-scale stochastic fractures following a predefined realistic distribution, which in principle can be derived from real data if available. The deterministic fractures are resolved in the fully coupled fracture-poroelasticity modeling, providing DFN-controlled inputs pivotal for the subsequent seismicity modeling and stress updating accounting for all fractures. Compared to the prevalent fracture-free, coupling-free and diffusion-only class of statistical models of induced seismicity and most other poroelastic models resolving fractures very limited in number and distribution, our method demonstrates great advantages as it not only produces realistically clustered distribution of seismicity with more sophisticated spatial-temporal characteristics (including e.g., doublet and multiplet), but also goes beyond the scope of most current models and provides a synthetic catalog of induced events, therefore allowing for the analysis of seismic source characteristics and the establishment of a connection between seismic observations and model physics.

Main findings from this study are:

1. The competition between the poroelastic stress compensation and the seismicity-induced stress loss is a viable mechanism for induced repeating events; it can
drive a favorably-oriented fracture through several seismic cycles within the time scale relevant to the problem.

(2) The distribution of induced seismicity is highly heterogeneous in space. It is not coincident with especially the fluid overpressure (which the diffusion-only class of models fail to explain); instead, the events are clustered near fractures that are well-oriented or subwell-oriented with respect to the initial stress field and in the meantime subjected to sufficient excess poroelastic shear stress. This highlights the importance of accounting for the interactions among fractures, the initial stress state and the poroelastic coupling.

(3) The induced events exhibit complex spatial-temporal characteristics. While a positive correlation is observable for events occurring along large-scale fractures, this trend immediately breaks down for events occurring in the surrounding matrix containing smaller-scale fractures of various orientations, owing to the highly complex spatial-temporal evolution of the poroelastic stress. Additionally, in the R-T space, the distribution of seismicity is also highly heterogeneous.

(4) In space only, a smooth front of either the fluid pressure or the seismicity can no longer be defined, due to the presence of the DFN and the associated high degree of heterogeneity. Nonetheless, in the R-T space, a front can be delineated qualitatively. Our modeling shows that the ‘observable’ front of the excess shear stress exceeds that of the fluid overpressure, which further exceeds that of the seismicity. In addition, due to the contribution of fractures, the modeled seismicity front is far beyond that predicted by a diffusion-only model utilizing only the hydraulic diffusivity of the porous matrix.

(5) The fractures induced to slip undergo highly nonlinear stress paths towards failure, which can only be modeled by including the poroelastic coupling.

(6) The magnitude of the induced events exhibits no spatial nor temporal dependence, owing to the random distribution of fractures and despite their lengths follow a realistic power-law distribution. Given the scale of our numerical example, the modeled distribution of the stress drop agrees very well with that of typically observed micro-earthquakes. On a fracture, the stress drop generally does not reach the maximum likely stress drop.

(7) Despite that the overall DFN obeys a realistic power-law decay in fracture length, the activated subset of the DFN may no longer obey the same scaling law, owing to that only favorably-oriented fractures are induced to slip. This might explain the commonly observed deviation in the $b$-value for induced seismicity.

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APPENDIX

The key equations used in calculating the source parameters of the induced events are shown here. First, $M_0$ can be calculated from the fracture dimension and the recorded $\Delta \tau$. Depending on the fracture geometry and the faulting regime, various formulas are available. Here, we opt for the one suitable for a rectangular dip-slip fracture (Kanamori and Anderson, 1975):

$$M_0 = \frac{\pi(\lambda + 2\mu)}{4(\lambda + \mu)} \Delta \tau W^2 L$$  \hspace{1cm} (19)

where $W$ is the fracture width (in the following numerical example under a plane strain condition, assumed as 1 m), $\lambda$ and $\mu$ are the Lamé’s constant and the shear modulus of the medium.

Second, $M_w$ is calculated from $M_0$ following (Hanks & Boore, 1984):

$$M_w = \frac{2}{3}(\log M_0 - 9.1)$$  \hspace{1cm} (20)

Finally, we adopt the following scaling laws that directly relate the permeability changes on a fracture to the event magnitude (Ishibashi et al., 2016):

$$k_{\perp} / k = 116.4 \times 10^{0.46M_w}$$

$$k_{//} / k = 13.1 \times 10^{0.46M_w}$$  \hspace{1cm} (21)

where $k_{\perp}$ and $k_{//}$ are the fracture permeability orthogonal and parallel to the direction of slip, and $k$ is a reference permeability of the fracture prior to slip and is related to the fracture length via a power scaling law. We note that other methods for mapping permeability changes from induced seismicity data are available (e.g., Fang et al., 2018).

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